Nonperturbative corrections to the quark self-energy

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Abstract

Nonperturbative self-energy of a bound quark is computed gauge-invariantly in the framework of background perturbation theory. The resulting Δm_q^2 is negative and is a universal function of string tension and current quark mass. The shift of hadron mass squared, M^2, due to Δm_q^2 is negative, large for light quarks, and solves the long-standing problem of Regge intercepts in relativistic quark models.

1 Introduction

The quark self-energy (QSE) operator for a quark propagating through the vacuum background field can be introduced in a gauge-invariant way, when quark is a part of a white object. Then one associates QSE with a part of the total energy of the bound state which does not depend on characteristics of other constituents, but depends on the given quark motion. In contrast to the perturbative QSE which is gauge and renormalization-scheme dependent [1] one can introduce the nonperturbative QSE in the framework of the Background Perturbation Theory (BPTh) [2, 3], in which case QSE can be expressed through the gauge invariant field correlators (for a general formalism of field correlators see [4, 5]).

It is useful to separate the nonperturbative contributions to QSE into two parts; one, coming from the color quark charge interaction with the nonperturbative background, and another created by the color magnetic moment of the quark (analogs of diamagnetic and paramagnetic contributions).

It was shown in [4, 5] that the quark charge part is responsible for creation of confining interaction hence is nonlocal and should be attributed to the $q\bar{q}$ interacting kernel. Only small distance-independent corrections can be separated out of this part and associated either with QSE or with $q\bar{q}$ interaction.

The color-magnetic moment part of QSE is instead large and independent of the string and antiquark motion (in the limit of small correlation length T_g of field correlators). It gives the dominant part of QSE and has specific negative sign, decreasing the hadron mass. In what follows we shall compute only this part and shortly discuss corrections from the charge part in the conclusions.

The next step is to calculate the contribution of QSE to the hadron mass and Regge trajectories. To this end one can use the Feynman-Schwinger (world-line) representation [6]-[9] and following from it the Hamiltonian einbein technic, introduced in [10, 11] (for reviews see lectures [9, 12] and recent paper [13]). As a result one obtains a negative

self-energy contribution from each quark and antiquark depending only on string tension and current quark mass.

Taken as a correction to the total hadron mass, this selfenergy contribution is favoured from phenomenological point of view. First of all it explains sign and magnitude of negative constant which is always added to the relativistic quark Hamiltonian in most existing models. In a similar way is resolved the old problem of Regge intercepts $J(M^2 = 0)$, when intercepts come out too low when computed through string tension without introduction of that phenomenological constant. Some examples for meson and baryon masses are given and the net effect for the Regge slope is discussed. The nontrivial character of the hadron mass shift obtained below is that it corrects the Regge intercept and keeps at the same time the value of the Regge slope at the standard string value $2\pi\sigma$. (One should note that simply adding a constant to the hadron mass, as it is usually done phenomenologically, would spoil the universality of Regge slope).

The plan of the paper is as follows. In section 2 the quark Green's function in the framework of BPTh is introduced and the world-line integral is used to separate contributions of color charge and color magnetic moment of the quark. In section 3 the lowest order (Gaussian) contribution to QSE is written down and expressed through the Gaussian correlators. In section 4 self-energy corrections to the Hamiltonian are derived and estimated, and the problem of Regge intercepts is discussed, while section 5 is devoted to conclusions and outlook.

2 Quark Green's function in vacuum background fields

Gluons play two different roles in QCD, namely, they create gluon condensate and string between quark and antiquark on one hand, and also propagate in the nonperturbative vacuum as valence gluons. Therefore it is convenient following [2, 3] to separate the total gluonic field A_μ into background NP field B_μ and field of valence gluons a_μ,

$$A_\mu = B_\mu + a_\mu \tag{1}$$

and using the 'tHooft identity to write the QCD partition function as

$$Z(J) = \frac{1}{N}\int e^{-S_E(A)+\int J_\mu A_\mu dx} DAD\psi D\bar{\psi} =$$

$$= \frac{1}{N'}\int DB\eta(B)e^{JBdx}\int DaD\psi D\bar{\psi}e^{-S_E(B+a)+\int J_\mu a_\mu dx} \tag{2}$$

with $N' = N\int DB\eta(B)$. Here $\eta(B)$ is an arbitrary measure and can be put $\eta(B)\equiv 1$ or chosen in such a way as to improve convergence of perturbation series in ga_μ. For what follows the value of $\eta(B)$ and explicit details of averaging over fields B_μ are not important.

According to (2) one can write any vacuum average as

$$\langle F(A)\rangle_A = \langle\langle F(B+a)\rangle\rangle_{B,a} \tag{3}$$

The Euclidean quark Green's function $G_q(x,y)$ can be written using the Feynman-Schwinger (world-line) Representation (FSR) [6]-[8] as

$$G_q(x,y) = (m_q+\hat{D})^{-1}_{x,y} = (m_q-\hat{D})_x(m_q^2-\hat{D}^2)^{-1}_{x,y} =$$

$$= (m_q - \hat{D})_x \int_0^\infty ds (Dz)_{xy} e^{-K} P_A \exp(ig\int_y^x A_\mu dz_\mu) P_F \exp(\int_0^s g\sigma F d\tau), \qquad (4)$$

where P_A, P_F are ordering operators of matrices A_μ and $F_{\mu\nu}$ respectively, $\sigma F \equiv \sigma_{\mu\nu}F_{\mu\nu} = \begin{pmatrix} \boldsymbol{\sigma}\mathbf{B} & \boldsymbol{\sigma}\mathbf{E} \\ \boldsymbol{\sigma}\mathbf{E} & \boldsymbol{\sigma}\mathbf{B} \end{pmatrix}$, and $\sigma_{\mu\nu} = \frac{1}{4i}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$, $\quad K = m_q^2 s + \frac{1}{4}\int_0^s \dot{z}_\mu^2 d\tau$.

In (4) the quark moving along its trajectory $z_\mu(\tau)$ is interacting with external total field A_μ by its color charge (the first exponent) and by its color magnetic moment (the second exponent in (4)). As we shall see it is the second interaction which yields the most important effect in creating the nonperturbative mass shift of the quark, and we shall calculate it in terms of field correlators. One should also note that the term ga_μ in $\hat{D} = \gamma_\mu(\partial_\mu - iga_\mu - igB_\mu)$ produces usual perturbative quark mass correction (including anomalous dimension for the mass evolution) and was investigated thoroughly especially for heavy quarks , (see [14] and references therein), and we shall not enter in discussion of this topic. Instead we shall concentrate on the contribution of nonperturbative background B_μ.

The Green's function (4) is not gauge-invariant. To define quark mass corrections in a gauge-invariant way, one must use the Green's function of a white object – of a meson or baryon. In the former case one can write [15] (in the flavour-nonsinglet case and neglecting sea-quark determinant)

$$G_M(x,y) = \langle tr \Gamma_1 G_q(x,y) \Gamma_2 G_{\bar{q}}(x,y)\rangle_B =$$

$$= \langle tr\{\Gamma_1 (m_{q_1} - \hat{D}) \int_0^\infty ds_1 Dz \int_0^\infty ds_2 D\bar{z} e^{-K_1 - K_2} \Phi_\sigma(x,y) \times$$

$$\times \Gamma_2 (m_{q_2} - \hat{D}) \Phi_\sigma(y,x)\}\rangle_B. \qquad (5)$$

Here $\Phi_\sigma(x,y)$ is the product of the last two exponents on the r.h.s. of (4), where the field A_μ is replaced by B_μ (setting $a_\mu \equiv 0$): $\Phi_\sigma(x,y) = P_B P_F \exp[ig\int_y^x B_\mu dz_\mu + \int_0^s g\sigma F d\tau]$.

One can note that the integrand of (5) contains the closed Wilson loop with insertions of operators $\hat{D}$ (twice) and operators σF (infinitely many times). Therefore the whole construction in (5) is gauge invariant. The sign of trace in (5) implies summing over Lorentz and color indices, and $\Gamma_i (i = 1, 2)$ stands for current vertices, $\Gamma_i = 1, \gamma_5, \gamma_\mu, ...$

3 Quark self-energy correction due to nonperturbative background

Consider the FSR for the quark Green's function (4). In the 2-nd order of perturbative expansion it can be written as

$$G_q(x,y) = (m_q - \hat{D}) \int_0^\infty ds \int_0^s d\tau_1 \int_0^{\tau_1} d\tau_2 e^{-K} (Dz)_{xu} d^4u (Dz)_{uv} d^4v (Dz)_{vy} \times$$

$$(igA_\mu(u)\dot{u}_\mu + g\sigma_{\mu\nu}F_{\mu\nu}(u))(igA_\nu(v)\dot{v}_\nu + g\sigma_{\lambda\sigma}F_{\lambda\sigma}(v)), \qquad (6)$$

where we have used the identities

$$(Dz)_{xy} = (Dz)_{xu} d^4u (Dz)_{uv} d^4v (Dz)_{vy}, \qquad (7)$$

$$\int_0^\infty ds \int_0^s d\tau_1 \int_0^{\tau_1} d\tau_2 f(s,\tau_1,\tau_2) = \int_0^\infty ds \int_0^\infty d\tau_1 \int_0^\infty d\tau_2 f(s+\tau_1+\tau_2,\tau_1+\tau_2,\tau_2) \quad (8)$$

At this point one can expand the quark Green's function only in color magnetic moment interaction (σF), which is useful when spin-dependent interaction can be treated perturbatively, as it is in most cases for mesons and baryons [12] (the exclusions are Goldstone bosons and nucleons, where spin interaction is very important and interconnected with chiral dynamics). In this case to the second order in (σF) one obtains

$$G_q^{(2)}(x,y) = (m_q - \hat{D}) \int_0^\infty ds \int_0^\infty d\tau_1 \int_0^\infty d\tau_2 e^{-m_q^2(s+\tau_1+\tau_2)-K_0-K_1-K_2} (Dz)_{xu} \times$$
$$\times \Phi(x,u) g(\sigma F(u)) d^4u (Dz)_{uv} \Phi(u,v) g(\sigma F(v)) d^4v (Dz)_{vy}. \quad (9)$$

In another way expanding $G_q = (m_q - \hat{D})(m_q^2 - D_\mu^2 - g\sigma F)^{-1}$ in powers of $(g\sigma F)$ it can be written as

$$G_q^{(2)}(x,y) = (m_q - \hat{D})(m_q^2 - D_\mu^2)^{-1}_{xu} d^4u \; g(\sigma F(u))(m_q^2 - D_\mu^2)^{-1}_{uv} d^4v \times$$
$$\times g(\sigma F(v))(m_q^2 - D_\mu^2)^{-1}_{vy}. \quad (10)$$

One can easily find the correspondence between (9) and (10) since $(m_q^2 - D_\mu^2)^{-1}$ is the scalar Green's function having the FSR as follows

$$G(x,y) \equiv (m_q^2 - D_\mu^2)^{-1}_{xy} = \int_0^\infty ds (Dz)_{xy} e^{-K} \Phi_\sigma(x,y). \quad (11)$$

Now keeping terms $g(\sigma F)$ up to the second order one can find from (10) and (5) (leaving aside for simplicity the factors $(m_q - \hat{D})$ in (5)) that one has in (5) an expansion

$$tr\langle W\rangle_B + tr\langle P_F W g\sigma F(u) g\sigma F(v)\rangle_B + ... \quad (12)$$

where operators $g\sigma F$ are inserted inside Wilson loop in the properly ordered form due to the operator P_F, keeping gauge invariance.

Using cluster expansion for W [4] the last term in (12) was computed in the Appendix of [15] with the result

$$\langle tr P_F F_{\mu\nu}(u) F_{\rho\lambda}(v) W(C)\rangle = tr\{[\langle F_{\mu\nu}(u,x_0) F_{\rho\lambda}(v,x_0)\rangle -$$
$$- g^2 \int d\sigma_{\alpha\beta}(w) \langle F_{\mu\nu}(u,x_0) F_{\alpha\beta}(w,x_0)\rangle \int d\sigma_{\gamma\delta}(z) \langle F_{\rho\lambda}(v,x_0)_{\gamma\delta} F(z,x_0)\rangle] \langle W(C)\rangle\} \quad (13)$$

Here $F(x,x_0) = \Phi(x_0,x) F(x) \Phi(x,x_0)$, with $\Phi(x,y) \equiv P \exp ig \int_y^x B_\mu(z) dz$. It is convenient in (13) to choose the point x_0 (note that $\langle W(C)\rangle$ does not depend on x_0) on the line between points u and v, since in this case the first correlator in (13) takes the form, studied in [4]

$$g^2 \langle F_{\mu\nu}(u) \Phi(u,v) F_{\rho\lambda}(v) \Phi(v,u)\rangle =$$
$$= \hat{1}_{ab}\{(\delta_{\mu\rho}\delta_{\nu\lambda} - \delta_{\mu\lambda} - \delta_{\mu\lambda}\delta_{\nu\rho}) D(u-v) + \frac{1}{2}[\partial_\mu(h_\rho\delta_{\mu\lambda} - h_\lambda\delta_{\nu\rho}) +$$
$$+ \partial_\nu(h_\lambda\delta_{\mu\rho} - h_\rho\delta_{\mu\lambda})] D_1(u-v)\}, \quad h_\mu \equiv u_\mu - v_\mu, \quad (14)$$

where $\hat{1}_{ab}$ is the unit operator in color space.

In what follows we shall keep only the first term in Eq. (13) in the form ((14), since the second term describes interaction of the quark spin with the string world sheet between the quark and antiquark and cannot be clearly associated with self-energy (actually this term contributes to the spin-orbit interaction between quark and antiquark, see discussion in [15, 16]).

Multiplying (13), (14) with spin operators one has

$$g^2\langle\sigma F(u)\Phi(u,v)\sigma F(v)\Phi(v,u)\rangle = \hat{1}_{ab}\hat{1}_{\alpha\beta}\{8D(u-v)+\frac{3}{2}\partial_\nu(h_\nu D_1(u-v))\}. \tag{15}$$

Here $\hat{1}_{\alpha\beta}$ is the unit operator in bispinor indices.

¿From (10) one can see that the following combination occurs playing the role of quark selfenergy correction

$$\Lambda \equiv \int d^4(u-v)g^2\langle\sigma F(u)\Phi(u,v)\sigma F(v)\Phi(v,u)\rangle G(u,v) \tag{16}$$

where $G(u,v)=(m_q^2-D_\mu^2)^{-1}_{u,v,}$ is given in (11). Since $G(u,v)$ depends on the interaction of quark with the string world sheet (through $D_\mu=\partial_\mu-igB_\mu$), then in general the correction (16) cannot be unambiguously attributed to QSE. However, at this point one can use the important fact, that $D(u)$ and $D_1(u)$ are short-ranged functions with the gluonic correlation length $T_g\approx 0.2\div 0.3$ fm from lattice calculations in the cooled vacuum [17] and even smaller, if one extracts T_g from the gluelump masses [18]. Therefore integration in (16) is limited to small $|u-v|$ and in this region one can replace $G(u,v)$ by $G_0(u,v)$ (since G_0 yields singular terms $\sim\frac{1}{(u-v)^2}$, while background field corrections give nonsingular contributions of the order of $T_g^2\langle F^2\rangle$. Thus one can expect the accuracy of this replacement to be of the order of $g^2T_g^4\langle E_i^2\rangle\sim 10\%$). Replacing G by G_0 in (16) and using for D, D_1 exponential form found on the lattice [17], one obtains in this approximation from (16)

$$\Lambda_0=\int d^4w\{8D(0)e^{-\delta|w|}+\frac{3}{2}D_1(0)\partial_\nu(w_\nu e^{-\delta|w|})\}\times$$

$$\times\frac{m_qK_1(m_q|w|)}{4\pi^2|w|},\quad \delta\equiv\frac{1}{T_g}. \tag{17}$$

It is convenient to define an integral ($K_1(x)$ is the Mc Donald function)

$$\varphi(m_q,\delta)=\int_0^\infty w^2dwe^{-\delta w}K_1(m_qw) \tag{18}$$

which has analytic form for $\delta>m_q$

$$\varphi(m_q,\delta)=-\frac{3m_q\delta}{(\delta^2-m_q^2)^{5/2}}\ln\frac{\delta+\sqrt{\delta^2-m_q^2}}{m_q}+\frac{\delta^2+2m_q^2}{m_q(\delta^2-m_q^2)^2}. \tag{19}$$

In the opposite case, $\delta<m_q$, one has

$$\varphi(m_q,\delta)=-\frac{3m_q\delta}{(m_q^2-\delta^2)^{5/2}}\arctan\frac{\sqrt{m_q^2-\delta^2}}{\delta}+\frac{\delta^2+2m_q^2}{m_q(\delta^2-m_q^2)^2}. \tag{20}$$

Two limiting cases are of interest for what follows,

$$\varphi(m_q,\delta)|_{m_q\to 0}=\frac{1}{m_q\delta^2} \tag{21}$$

$$\varphi(m_q,\delta)|_{m_q\to\infty}=\frac{2}{m_q^3}-\frac{3\pi\delta}{2m_q^4}+O\left(\frac{\delta^2}{m_q^5}\right). \tag{22}$$

In a similar way one can define the integral in front of $D_1(0)$ in Eq. (17),

$$\varphi_1(m_q,\delta)\equiv(4+\delta\frac{\partial}{\partial\delta})\varphi(m_q,\delta)=$$

$$=\frac{15m_q^3\delta}{(\delta^2-m_q^2)^{7/2}}\ln\frac{\delta+\sqrt{\delta^2-m_q^2}}{m_q}-\frac{8m_q^4+9m_q^2\delta^2-2\delta^4}{m_q(\delta^2-m_q^2)^3};(\delta>m_q)$$

$$=-\frac{15m_q^3\delta}{(m_q^2-\delta^2)^{7/2}}\arctan\frac{\sqrt{m_q^2-\delta^2}}{\delta}+\frac{8m_q^4+9m_q^2\delta^2-2\delta^4}{m_q(m_q^2-\delta^2)^3};(\delta<m_q). \tag{23}$$

The limiting values of φ_1 are

$$\varphi_1(m_q,\delta)|_{m_q\to 0}=\frac{2}{m_q\delta^2} \tag{24}$$

$$\varphi_1(m_q,\delta)|_{m_q\to\infty}=\frac{8}{m_q^3}-\frac{15\pi\delta}{2m_q^4}+O\left(\frac{\delta^2}{m_q^5}\right). \tag{25}$$

Comparing the expansion of $(m_q^2+\Delta m_q^2-D_\mu^2)^{-1}$ with (10) one can define the QSE for $m_q\to 0$

$$\Delta m_q^2=-\Lambda_0=-\frac{1}{\delta^2}(4D(0)+\frac{3}{2}D_1(0)). \tag{26}$$

For purely exponential correlator $D(x)$ one can connect $D(0)$ with string tension σ; in the Gaussian approximation, which was checked recently using Casimir scaling arguments to be accurate within few percents [19, 20], one has

$$\sigma=\frac{1}{2}\int D(x)d^2x=\frac{\pi D(0)}{\delta^2}. \tag{27}$$

As a result one has for the self-energy if $m_q\to 0$

$$\Delta m_q^2=-\frac{4\sigma}{\pi}(1+\xi),\quad \xi=\frac{3}{8}\frac{D_1(0)}{D(0)}. \tag{28}$$

Lattice measurements [17] give $\xi<\frac{1}{8}$, and later we shall omit this term. We note at this point that Eq.(28) is universal in the sense that it does not depend on the exponential or any other form of correlator in (15), since the integral (18) for $m_q\to 0$ assumes the form of integral (27) for string tension.

For nonzero values of m_q one has instead of (28)

$$\Delta m_q^2(m_q)=-\frac{4\sigma}{\pi}\eta,\quad\text{with}\quad \eta=\frac{\varphi(m_q,\delta)}{\varphi(0,\delta)}. \tag{29}$$

For example, for the strange quark with $m_s=0.175$ GeV one has $\eta\cong 0.88$, while for the c-quark, $m_c\cong 1.7$ GeV, $\eta\cong 0.234$ and finally for the b-quark, with $m_b\approx 5$ GeV one obtains $\eta\cong 0.052$.

4 Self-energy corrections to Hamiltonian and hadron masses

Using expression (5) for the meson Green's function and exploiting the einbein formalism [10, 11] one can construct the relativistic Hamiltonian. We introduce the squared quark masses with corrections due to QSE obtained in the previous section, $m_q^2 \to m_q^2 + \Delta m_q^2$ and arrive at the expression

$$H = H_0 + \Delta H_Q + \Delta H_s,$$

$$H_0 = \sum_{i=1}^{2} \left(\frac{m_q^2(i)}{2\mu_i} + \frac{\mu_i}{2} \right) + \frac{p_r^2}{2\tilde{\mu}} + \frac{\hat{L}^2/r^2}{2[\mu_1(1-\zeta)^2 + \mu_2\zeta^2 + \int_0^1 d\beta(\beta-\zeta)^2\nu(\beta)]}$$
$$+ \frac{\sigma^2 r^2}{2} \int_0^1 \frac{d\beta}{\nu(\beta)} + \int_0^1 \frac{\nu(\beta)}{2} d\beta \tag{30}$$

and

$$\Delta H_q = \sum_{i=1}^{2} \frac{\Delta m_q^2(i)}{2\mu_i}. \tag{31}$$

Here we have defined the reduced dynamical mass $\tilde{\mu}$ and the parameter ζ:

$$\tilde{\mu} = \frac{\mu_1\mu_2}{\mu_1+\mu_2}, \quad \zeta = \frac{\mu_1 + \int_0^1 \beta\nu(\beta)d\beta}{\mu_1+\mu_2+\int_0^1 \nu d\beta} \tag{32}$$

and $m_q(1)$ and $m_q(2)$ are current quark and antiquark masses, renormalized at the scale around 1 GeV, while μ_1, μ_2 and $\nu(\beta)$ are einbein functions which should be found from the minimum of this Hamiltonian. For the discussion and practical use of this Hamiltonian including spin-depending part ΔH_s, beyond the original papers [11] see also lectures [12] and recent paper [13].

It is important that the dynamical masses μ_1, μ_2 are found from the stationary point analysis of the Hamiltonian and these values $\mu_1^{(0)}, \mu_2^{(0)}$ can be used to estimate self- energy corrections to the Hamiltonian and its eigenvalues.

Hence the eigenvalues of H will be shifted by an amount read off from (31)

$$\Delta H_q^{meson} = -\frac{2\sigma}{\pi} \left(\frac{1}{\mu_1^{(0)}} + \frac{1}{\mu_2^{(0)}} \right), \tag{33}$$

i.e. from each quark the shift is equal approximately to $-\frac{2\sigma}{\pi\mu_1^{(0)}}$. Taking into account that the lowest eigenvalue $M^{(0)}$ is equal (without color Coulomb interaction) $4\mu^{(0)}$ [10, 11, 13], one has for equal masses $\mu_1^{(0)} = \mu_2^{(0)} = \mu^{(0)}$

$$M^{(0)}(L = n_r = 0) + \Delta M = 4\mu^{(0)} - \frac{4\sigma}{\pi\mu^{(0)}}. \tag{34}$$

In a similar way one can write the shift of the baryon mass. Using [21, 22] one can write

$$\Delta H_q(baryon) = -\frac{2\sigma}{\pi} \sum_{\imath=1}^{3} \frac{1}{\mu_i^{(0)}}, \tag{35}$$

where $\mu_i^{(0)}$ are to be found again from the minimum of baryon Hamiltonian [21, 22]. In the lowest approximation (color Coulomb and spin interaction neglected) and for light quarks one has $\mu_i^{(0)} = \mu^{(0)}$ and $M^{(0)} = 6\mu^{(0)}$, so that the resulting corrected mass is

$$M = M^{(0} + \Delta H_q = 6\mu^{(0)} - \frac{6\sigma}{\pi\mu^{(0)}}. \tag{36}$$

The same type of correction occurs for hybrids, since ΔH_q has the same form (33) as for mesons, however the values of $\mu_1^{(0)}$ and $\mu_2^{(0)}$ are to be found for the total hybrid Hamiltonian (see [12]) containing in addition dynamical mass μ_g of the valence gluon.

We now come to the one of the most important problems of hadron spectrum, which to the knowledge of the author had before no solutions – the problem of Regge intercepts. It was shown in [11, 25, 26] that the Hamiltonian H_0 ensures the string Regge slope and the square of its mass eigenvalue is $M^2(L) = 2\pi\sigma L + C_0$, where C_0 is positive and too large (for $m_q = 0$, $C_0 \approx 11\sigma = (1.4 \text{ GeV})^2$ when Coulomb and hyperfine interaction is not taken into account), hence the Regge intercept

$$J(M = 0) = L + 1 = 1 - \frac{C_0}{2\pi\sigma} \tag{37}$$

is negative (-0.73), while for the experimental ρ trajectory it is positive and near +0.5. Therefore one needs such mass corrections which would shift Regge trajectories upwards in the Chew-Frautschi plot ($J(M^2)$ vs M^2) not changing its slope.

As we shall see now, the correction ΔH_q (30) has exactly this property. To start we take the corrected mass (36) and consider its square

$$M^2 \equiv (M^{(0)} + \Delta M)^2 = (4\mu^{(0)} - \frac{4\sigma}{\pi\mu^{(0)}})^2 = (M^{(0)})^2 - \frac{32\sigma}{\pi} + O(\Delta M)^2) \tag{38}$$

Thus to our accuracy (first order in ΔM) mass squared is shifted by a constant not depending on quantum numbers, $-\frac{32\sigma}{\pi}$, and if $(M^{(0)})^2$ corresponded to a linear Regge trajectory, the same will be true for the shifted trajectory with the same slope.

If one calculates $M^{(0)} = 4\mu^{(0)}$ from the simplified Hamiltonian, obtained from (31) by putting $\hat{L} \equiv 0$ and $p_r^2 \to \mathbf{p}^2$(this is very close to the relativistic quark model Hamiltonian $H_{RQM} \equiv 2\sqrt{\mathbf{p}^2 + m_q^2} + \sigma r$) then the slope of $M^{(0)}$ would be 8σ instead of expected string-like $2\pi\sigma$ [10], and this slope is kept intact by the shift (38). To resolve this inconsistency one should take into account the term with $\hat{L}^2$ in (31) which contains the moment of intertia of the rotating string and changes the slope to the correct value $2\pi\sigma$ (11).

Exact calculations with the Hamiltonian (31) require numerical solutions yielding the linear Regge trajectories with the slope very close to $2\pi\sigma$ (see [25, 26] for results and discussions). Below we shall consider the effect of string rotation as a correction $\Delta M^{(1)}$ [11] and write for the whole mass squared

$$M^2 \cong (4\mu^{(0)} + \frac{\Delta m_q^2}{\mu^{(0)}} + \Delta M^{(1)})^2 \tag{39}$$

with $\Delta m_q^2 = -\frac{4\sigma}{\pi}$ and $\Delta M^{(1)} = -\frac{32}{3}\frac{\sigma^2 L(L+1)}{(M^{(0)})^3}$. Denoting

$$(4\mu^{(0)} + \frac{\Delta m_q^2}{\mu^{(0)}})^2 = 8\sigma L + \bar{C}_0, \tag{40}$$

one obtains to the lowest order

$$M^2 \cong 8\sigma L + \bar{C}_0 - \frac{32}{3}\frac{\sigma^2 L(L+1)}{8\sigma L + \bar{C}_0}. \tag{41}$$

The Regge slope $\frac{\partial M^2}{\partial L}$ can be computed from (41) both for $L = 1$ and for $L \to \infty$, which yields $\frac{\partial M^2}{\partial L} = \frac{20}{3}\sigma(L \to \infty)$, $\frac{\partial M^2}{\partial L} \cong 6.5\sigma(L = 1)$.

One can see that both slopes are rather close to the string one, $2\pi\sigma$, while the self-energy term Δm_q^2 contributes only to the constant $\bar{C}_0$, shifting it towards the experimental value of m_ρ^2. Thus QSE indeed can solve the problem of Regge intercepts to a reasonable accuracy. To get an idea of the magnitude of ΔH_q, consider the ρ - meson, corresponding to the solution of the Hamiltonian (30) with $L = n_r = 0$. Taking $\sigma = 0.18$ GeV^2 one obtains [10, 12] $\mu_1^{(0)} = \mu_2^{(0)} = \mu^{(0)} = 0.352$ GeV and $M^{(0)} = 4\mu^{(0)} = 1.41$ Gev, while (33) yields $\Delta H_q(meson) = -0.65$ GeV, with the total mass $M = M^{(0)} + \Delta H_q = 0.76$ GeV. This last figure is close to the experimental ρ-meson mass, but our result should be corrected by hyperfine spin term and Coulomb interaction which tend mass in different directions with a net shift around or less than 0.1 GeV.

A similar situation occurs for baryons. Taking $\sigma = 0.15$ GeV^2 (see [23] and [24] for the phenomenologically motivated choice and its theoretical justification), one obtains from [22] for light quarks $\mu^{(0)} = 0.957\sqrt{\sigma}$, and $M_B^{(0)} = 6\mu^{(0)}$. As a result one obtains for Δ mass (without spin and Coulomb correction)

$$M_B^{(0)} + \Delta M = (2.22 - 0.77)\text{GeV} = 1.44\text{GeV}.$$

For strange baryon, Ω, one has instead for $m_q(s) = 0.175$ GeV the dynamical mass μ_s =0.415 GeV [22] and finally

$$M^{(0)}(\Omega) + \Delta H_q = (2.46 - 0.607)\text{GeV} = 1.85\text{GeV}.$$

These examples are only to illustrate the order of magnitude of resulting corrections. We leave the problem of realistic meson and baryon mass calculations with spin and Coulomb interaction included to future publications.

5 Discussion and conclusions

In most model calculations of hadron masses done heretofore the absolute values of masses have been defined up to a constant, varying from one family of hadrons to another. Also in our previous calculations [12, 13] this constant was introduced and associated with the quark self-energies, however the latter were defined phenomenologically [13] to be around -0.25 GeV per light quark, but not derived from theory.

In the present paper we have looked into problem of quark self-energies and absolute values of hadron masses, and calculated both in terms of the only scale of nonperturbative QCD - the string tension σ.

As a result we have succeeded in several respects. Firstly, all our theory is based on quark current masses, assumed to be renormalized at the scale of around 1 GeV, where NP effects do not yet practically enter. These masses should be associated with the pole masses, introduced in case of heavy quarks (see e.g. [14]).

Secondly, QSE corrections yield a numerically reasonable shift of masses downwards, and for light quark this shift is indeed around -0.25 to -0.3 GeV, as was required by phenomenology [13].

Thirdly, and this is most important featurefrom theoretical point of view, this QSE shift does not spoil the correct Regge slope and at the same time resolves the old problem of Regge intercept which was too low in all previous calculations (if negative constant is not introduced).

Let us now discuss the nature of this correction and compare it to another possible approach. First of all, the negative sign of QSE is evidently connected to the paramagnetic mechanism, which is at work here. One should note that the quark charge interaction, present in (4) in the form of gA_μ, produces effect of another (positive) sign to the quark mass squared, and the net result is bounded from below, since in $(m^2-\hat{D}^2)^{-1}$ the operator $-\hat{D}^2=-D_\mu^2-g(\sigma F)$ is nonnegatively defined, and vanishes only for quark zero modes. However in the confining phase the charge part gA_μ produces for the white system of $(q\bar{q})$ the confining interaction σr and belongs to the whole worldsheet of the string and cannot be associated with QSE. Instead the part $g(\sigma F)$ contains the local correction (in the limit of small T_g) $g^2\langle(\sigma F)(\sigma F)\rangle$, which can be treated as QSE and was computed in the present paper. One important conclusion is that actually QSE should be considered finally as a correction to the total hadron mass, and not to the mass of the quark, since the latter has no definite meaning inside the hadron. Therefore one should not give much sense to the fact that for light quarks the total quark mass $m_q^2+\Delta m_q^2$ is negative –one should have in mind that this contribution is only a part of the Hamiltonian (30) and should be computed as a correction to its eigenvalues. (Note that one cannot include ΔH_q in H_0 to find $\mu_i^{(0)}$ from stationary point analysis, since this procedure would go beyond the lowest order in ΔH_q, while ΔH_q was itself computed to the lowest order only).

To get more understanding of negative QSE correction to the quadratic (in Dirac operator $\hat{D}$) Hamiltonian, one can consider masses of heavy-light mesons $M_{HL}=M_Q+\varepsilon$ computed in two different approaches, namely to compare eigenvalues of this quadratic Hamiltonian in the case when one mass, $m_q^{(2)}\equiv M_Q$, is very heavy, to the eigenvalues of the corresponding Dirac equation with linear confining interaction. Taking the latter from [27, 28] one has for lowest Dirac eigenvalues,

$$\varepsilon_0^{(D)}=1.62\sqrt{\sigma} \tag{42}$$

while for the Hamiltonian $H_0(m_q^{(2)}\to\infty)$ one obtains [13]

$$\varepsilon_0^{(H)}=2.26\sqrt{\sigma}. \tag{43}$$

One can see the difference of about 0.3 GeV, which is very close to the mass shift due to QSE. This is not surprising since in $\varepsilon_0^{(D)}$ the corresponding QSE correction is absent, while for $\varepsilon_0^{(H)}$ this correction is present and is given by $\frac{\Delta m_q^2}{2\mu}$. Therefore working in the linear Dirac formalism like that developed in [29] one should not include the aforementioned QSE correction and the Regge-intercept problem does not appear. Finally it is interesting to note, that self-energy correction similar to the one considered in the present paper appears in the 1+1 QCD, where in 't Hooft equation [30] each quark obtains a negative contribution to the mass squared equal to $-2\sigma/\pi$, i.e. one half of self-energy correction computed above in 3+1 QCD.

Summarizing, we have calculated the nonperturbative QSE correction explicitly and found the shift in hadron masses and Regge intercept which is strongly favored phenomenologically.

Results of more detailed analysis for both light and heavy hadrons are to be published elsewhere.

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